

Urban Malignancy: Similarity in the Fractal Dimensions of Urban Morphology and Malignant Neoplasms

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Abstract

All contemporary landscapes on the planet feature the aggressive growth of cities and other urbanizations. In 1990, we suggested that the similarity between urban forms and malignant lesions could be studied with the use of fractal geometry. Two separate disciplines have emerged since then: the study of urban morphology using various fractal analyses, and “oncological mathematics,” the study of malignant lesions using fractal analysis. Several mathematical techniques are used in both fields to conduct these studies. From the point of view of a physician, the expanding, invasive, colonizing urban form with highly irregular borders resembles a malignant lesion. Malignant neoplasms have at least four major characteristics: rapid, uncontrolled growth; invasion and destruction of adjacent normal tissues (ecosystems); metastasis (distant colonization); and de-differentiation. Many urban forms are almost identical in general appearance, a characteristic that would qualify as “de-differentiation.” Large urban settlements display “rapid, uncontrolled growth” expanding in population and area occupied at rates of from 5 to 13% per year. We propose a null hypothesis that “there is no similarity or correlation in the fractal dimension of urban forms and malignant neoplasms.”

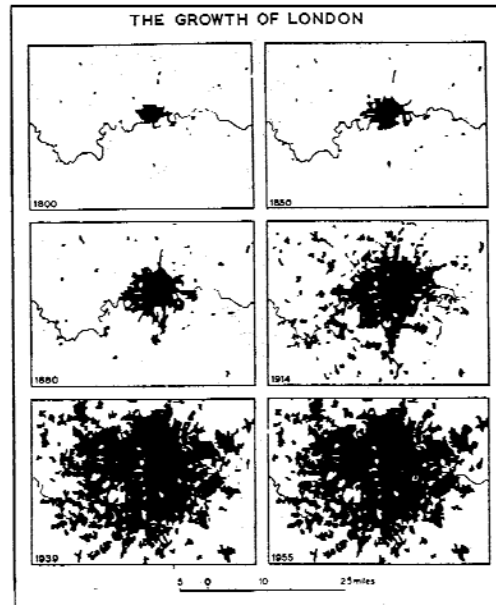
Introduction

Why do cities and towns grow in the way they do? What are the laws, if any, governing the growth patterns of urban settlements? Urban geographers have sought to answer these questions for decades.

Everywhere in the world, regardless of culture or nationality, modern human settlements of all sizes have demonstrated rapid growth over the past two centuries characterized by steady expansion of the margins, the development of new communities at nodes distant from the urban center, and the replacement of natural and cultivated landscapes by urban structures.

As distinguished from ancient cities with walls circumscribing the urban entity, the borders of human settlements of all sizes have become ill-defined except for natural barriers such as rivers, cliffs, and seashores. There is an explosive, ragged, invasive appearance that increases with time, as seen in the growth of London from 1800 to 1950, for example (Fig. 1).

Fig. 1 London, 1800 – 1950 (from Johnson, 1972)

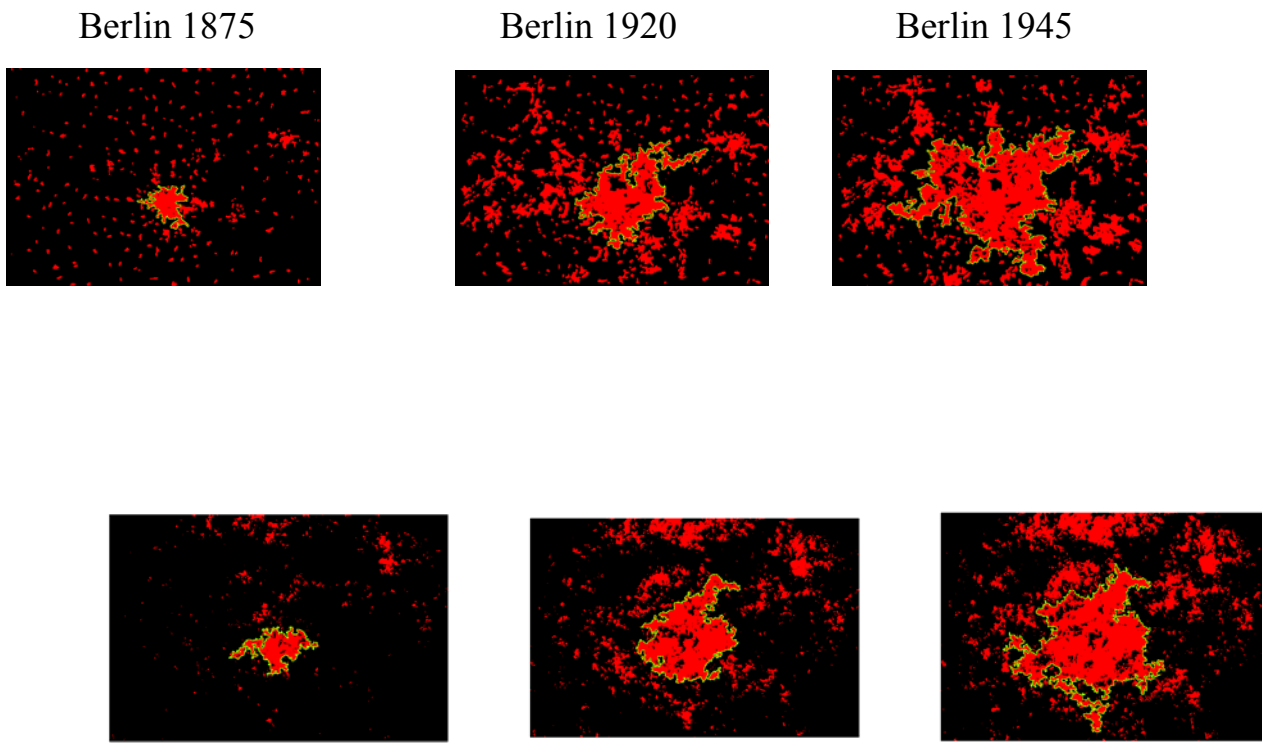


These patterns are universal and found on every continent. They are independent of scale, culture, social organization, and geography (Frankhauser 1998 p 137; Thomas, et al 2007, Table 3; Frankhauser, 2004; Manrubia, et al, 1999).

Fractal patterns in cities and malignancies

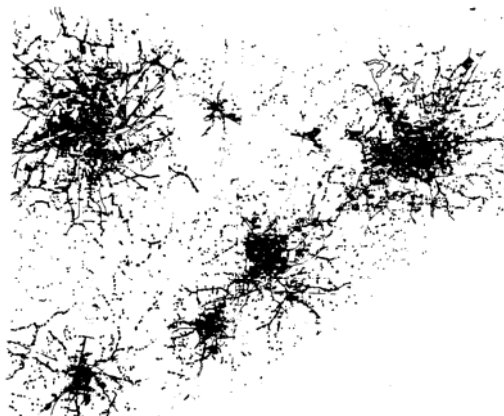
The exposition of fractal geometry by Mandelbrot (1983) provided a critical tool for the study of urban form, and since 1990, the discipline of urban geography has developed a large literature with an emphasis on the use of fractals to study these complex irregularities. Mathematical modeling of fractal growth has been used to simulate and understand urban growth patterns (White and Engelen, 1993; Andersson, et al, 2002a; Andersson, et al 2002b; Onural, 1991; Paget, 1999; Batty, 2005). Makse, et al (1995; 1998) used correlated percolation simulation to produce a pattern similar to the growth of Berlin from 1850 through 1945 (Fig 2).

Fig. 2 Berlin 1875 – 1945 compared to simulated correlated percolation (Makse, et al, 1995)



To the eye of a physician trained in basic pathology, the appearance and growth patterns of urban settlements either at an instant in time or over time resembles the form and growth patterns of a malignant lesion. An example that illustrates this is the five-city cluster in North Carolina (Fig. 3) from 1958 found in Chapin and Weiss' *Urban Growth Patterns* (1962).

Fig. 3 Five city cluster, North Carolina (Chapin and Weiss, 1962)



In the first edition (1950) of Gallion and Eisner's book, *The Urban Pattern*, this same morphology was shown to be present in the growth patterns of London, Kansas City, Chicago, New York City, Barcelona, and Zurich.

Malignant neoplasms in organisms have several classical characteristics: a) rapid, uncontrolled growth; b) invasion and destruction of adjacent normal tissues; c) metastasis (distant colonization); and de-differentiation (loss of characteristic cell and tissue appearance unique to each kind of tissue) (Anderson, 1961; Perez-Tamayo, 1961). Metastatic lesions tend to be more aggressive and grow faster than the original tumor.

In 1990, we proposed that the similarity in the morphology of urban settlements and malignant lesions could be studied with the use of fractal geometry (Hern, 1990). We placed images of malignant lesions and urban settlements side-by-side to show this similarity (Fig. 4).

Fig. 4. Comparative images, malignant lesions and urban settlements (Hern, 1990)



a) pulmonary adenocarcinoma; b) metastatic malignant melanoma; c) Baltimore; d) metastatic malignant melanoma; e) five-city cluster, North Carolina

Mandelbrot (1983) began his introduction of fractal geometry by pointing out that most forms in nature do not conform to Euclidean geometry based on straight lines and smooth curves. The topological dimension of a point is zero, the topological dimension of a straight line is 1, and the topological dimension of a surface is 2. But the dimension of the edge of an irregular, fragmented object (such as the coastline of Britain) is a fraction somewhere between 1 and 2. It is a “fractal pattern.” The famous question, “How long is the coastline of Britain?” is answered by the statement, “It depends on the length of your ruler.” If the unit of measurement ℓ is 1 kilometer, you get one measurement, but if the unit of measurement ℓ is 1 centimeter, you get a longer measurement. “The number of steps (L) multiplied by ℓ is an approximate length $L(\ell)$.”¹ The length $L(\ell)$ of the subject approaches infinity as the unit of measurement approaches zero ($\ell \rightarrow \lim_{\ell \rightarrow 0} \infty$).

This problem is resolved by imposing a series of grids of boxes of different sizes on the outline being studied and counting the number of $N(r)$ squares of side length r containing a portion of the edge in question. The logarithm of $N(r)$, the number of occupied cells, versus the logarithm of $1/r$, where r is the size of one cell, gives a [regression] line whose gradient corresponds to the box dimension (Morency, et al, 2003).

This “box-counting” technique is common to all studies of fractal dimension (D) as compared to the topological (Euclidean) dimension (D_T). When the measuring unit is

extremely small, $D \approx D_T$. The fractal dimension D of a chessboard or a circle surface is 2, the same as the topological dimension D_T , but the fractal dimension of either of these two-dimensional surfaces with part of it missing is between 1 and 2 ($1 < D < 2$). D_T is always an integer (0,1,2,3..) but D need not be an integer.² The fractal dimension D of the coastline of Britain, for example, is approximately the same as a triadic Koch curve where $D = \log N / \log (1/r) = \log 4 / \log 3 \sim 1.2618$ (Mandelbrot, 1983, p. 36).

Fractal geometry is a way of comparing irregular forms, even at different scales since the approximate measure of the dimension D is independent of the unit of measurement (Mandelbrot, 1983, p 30).

There has been a steady development in the use of fractal geometry and mathematical models of fractal growth in the study of malignant tissues. The same methods used in urban geography have been used to study malignant processes from the cellular to the macro level of tissue architecture (Aribisala and Claridge, 2005; Weyn, et al, 1999; Sedivy, et al, 1999). These studies have been highly productive, particularly in the understanding and characterization of solid tumors, and they have been used to monitor treatment of various malignancies. (Metser, 2004; Norton, 2005; Dimitrakopoulou-Strauss, et al, 2003).

Border irregularity, especially in skin and solid tumors, has been regarded as a key to the degree of malignancy (Keefe, et al, 1990) The probability of a lesion being malignant increases with increased fractal dimension as a result of border irregularity (Claridge, et al 1992; Landini and Rippin, 1999).

Malignant melanoma, a highly aggressive pigmented skin cancer, is most easily characterized by its visibly irregular border. This has been studied by Lee and Claridge (2005) to show that the greater the degree of border irregularity, the more malignant the lesion. A malignant melanoma was 8.9 times more likely to have an irregular border than a benign lesion. The study confirmed that border irregularity is an important clinical feature for the diagnosis of malignant melanoma. (Fig. 5)

Fig. 5 *Malignant melanoma*



Metser (2004) found that metastases of colorectal cancer with fractal dimensions greater than 1.35 were associated with poor survival rates.

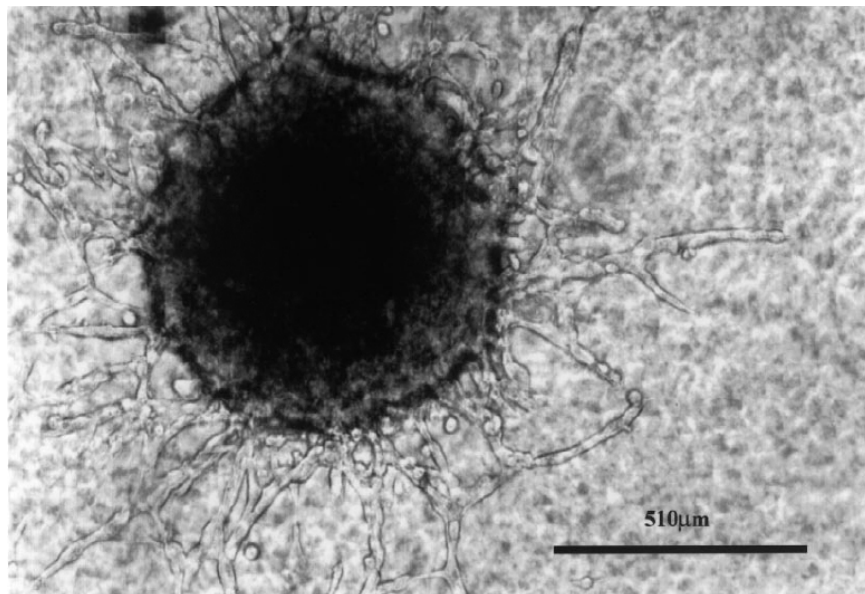
Dimitrakopoulou-Strauss et al (2001) reported that fractal dimension among other parameters accounted for significant differences in diagnostic pairs of tissues and that 84 % of GIII soft-tissue sarcomas were classified correctly.

In studying the fractal dimensions of the epithelial-connective tissue interfaces in premalignant and malignant epithelial lesions of the floor of the mouth, Landini and Rippin (1993) found that lowest FD value found was 0.99 (a normal mucosa) and the highest 1.61 (a carcinoma).

At the cellular level, cancer cells show a different level of fractal dimension than normal cells. In studying a frequency distribution histogram of fractal cell dimensions, Bauer and Mackenzie (2001) found that a large proportion of the lymphocytes of hairy-cell leukemia patients had a fractal dimension exceeding 1.28, whereas none of the lymphocytes of healthy persons exceeded a fractal dimension of 1.28.

Kansal, et al, (2000) developing a 3-dimensional cellular automaton simulation of glioblastoma multiforme, a uniformly fatal type of brain cancer, showed a “spheroid” from an actual cancer, which, at 0.5 cm in diameter contained approximately 1,000,000 cells representing about 20 doublings of the original malignant cell or cell mass (Fig. 6)

Fig. 6. Spheroid, malignant glioblastoma multiforme (200x) from Kansal, et al (2000)



The spheroid displays a highly invasive and irregular border. A three-dimensional simulation followed the path of the clinical lesion closely. Mathematical insertion of a “cell” with a faster doubling time showed the aggressive domination of the more rapidly growing tissue. The authors concluded that the growing tumor must be investigated and understood as a “self-organizing complex dynamic system.” The behavior and process of

the spheroid, or cell mass, is not explained by the behavior or morphology of the individual cells.

Kikuchi, et al, (2004) found that the fractal dimensions of surface growth patterns in different grades of endometrial adenocarcinoma ranged from 2.318 to 2.383. These values were greater than the topological dimension of a surface ($= 2$), reflecting a three-dimensional structure. The authors concluded that the endometrial adenocarcinoma has a fractal structure, and that the mean fractal dimension may differ according to histologic grades

Just as a solid tumor has a 3-dimensional fractal property, a modern city with skyscrapers and underground transportation networks has a fractal dimension that would exceed 2. The surface outline of the city, however defined, would have a fractal dimension between 1 and 2, the silhouette of the city skyline would have a fractal dimension of between 1 and 2, the silhouette of the underground installations would have a fractal dimension between 1 and 2, and the entire city in its extended border and vertical features, both above and below the surface, would have a fractal dimension of between 2 and 3 (Klonowski, 2000; Frankhauser, 1998). (Fig. 7)

Fig 7. São Paulo, Brazil, 2001



Image: Saopaulo copan.jpg

In their study of Belgian communities, Thomas and Frankhauser observed fractal dimensions of between 1.5 and 1.8 for seven communities (Thomas, et al, 2007). Batty and Longley show similar fractal dimensions for the outlines of 28 cities (1994; Table 7.1, p. 242). Fractal dimension has a tendency to increase as a city grows. Morency and Chapleau (2003) found that the fractal dimension of the urban outline of Montreal increased from 1.43 to 1.63 from 1945 to 1996.

Significance of fractal dimension

Just because an object has a large fractal dimension or displays fractal growth does not have any meaning in itself, nor does the comparison of fractal dimensions between two objects. An artichoke has a large fractal dimension, and it is not malignant. Similarities in the fractal dimension of cities and cancers are, at best, superficial. If that were the only similarity, it would not be meaningful. The strong evidence of fractality in both cities and cancer only calls attention to other possibilities.

Cities, however, have, at the superorganic scale, all the four major and classical characteristics of a malignant process: rapid, uncontrolled growth; invasion and destruction of adjacent normal tissues (space or ecosystems); metastasis (distant colonization); and de-differentiation (Anderson, 1961). The latter is seen in the comparison of the morphologies of major cities as well as in their growth patterns over time. The city outline of London is very similar to the outlines of Berlin, Houston, São Paulo, Denver, Brasilia, or other examples. Whereas the walled cities of medieval Europe and tribal settlements have had certain characteristic patterns that reflected the local culture, modern settlements have the same ragged, invasive, expanding appearance wherever they are found. This is a prime example of “de-differentiation.”

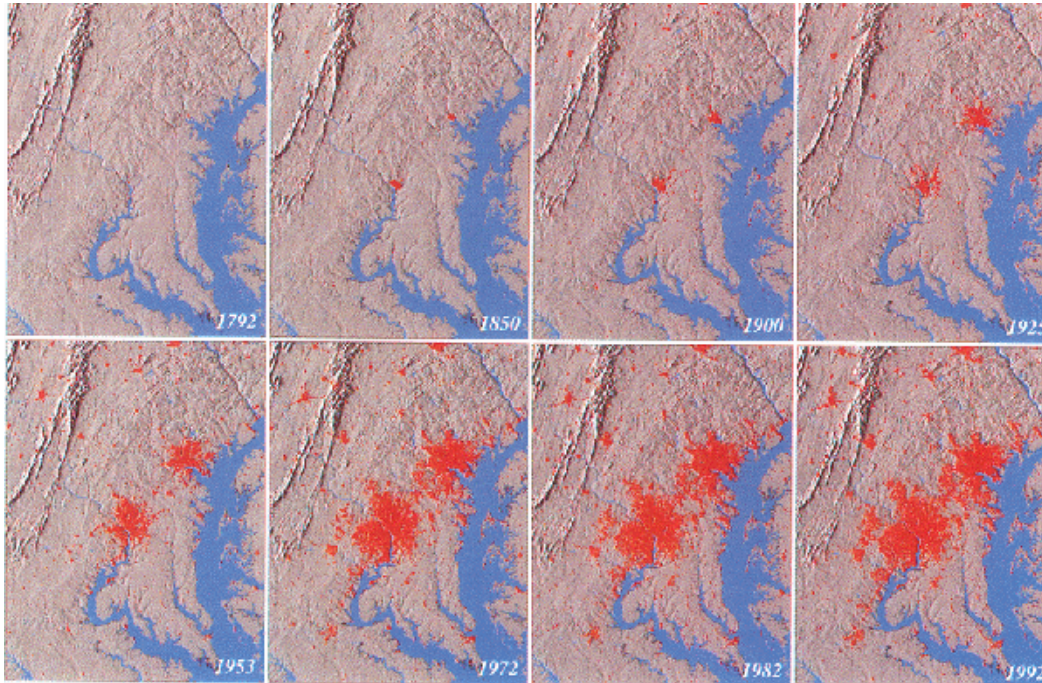
Satellite lesions / metastases / outlying communities

The development of satellite lesions or satellite communities (suburbs) is similar in both urban growth and malignant growth. The questions are similar in both cases. Does the malignancy send out metastases because it has become large, or does it become large because it coalesces with malignant cells seeded in tissues some distance from the center of the tumor (Norton, 2005)? Do peripheral urban settlements occur because the urban center has become large and congested, or do cities coalesce and “devour” small communities that have developed at certain “nodes” beyond the main city, (Benguigui, et al, 2000; Benguigui & Czamanski, 2004) or, as Frankhauser calls it, the “enslaving principle” (Frankhauser, 1998, p 141)? It is a similar process and a similar question.

Rapid urban growth

Although documentation of rapid urban growth shows this happening from the early 19th century, it is often described as having been “explosive” in the twentieth century (Thomas, et al, 2007). In Brazil, Kubitschek established Brasilia as the new national capitol in 1956, carving it out of the wilderness of Goiás. From the time of its founding through 1980, Brasilia grew at rates of from 13 to 9% per year (UN, 2005). That can only be characterized as “rapid, uncontrolled growth.” A study of LandSat photographs by Masek, et al (2000; Foresman, 2000) showed that the Washington-Baltimore metropolitan area was expanding at the rate of 22 square kilometers per year from 1973 through 1996. (Fig. 8)

Fig. 8 Growth of Baltimore-Washington Metropolitan Area 1792 – 1992



The town of Pucallpa, in the upper Peruvian Amazon, began as a colony of about 1500 Peruvians in 1943, these settlers having displaced the original indigenous Amazonian inhabitants. By 1946, when the trans-Andes highway had been completed, the population was about 3,000. It had a population of 30,000 in 1964, and it is now about 400,000 inhabitants.(Fig 9)

Fig. 9. Pucallpa, Peru, 1989 (from NASA LandSat files, courtesy of Compton J. Tucker)

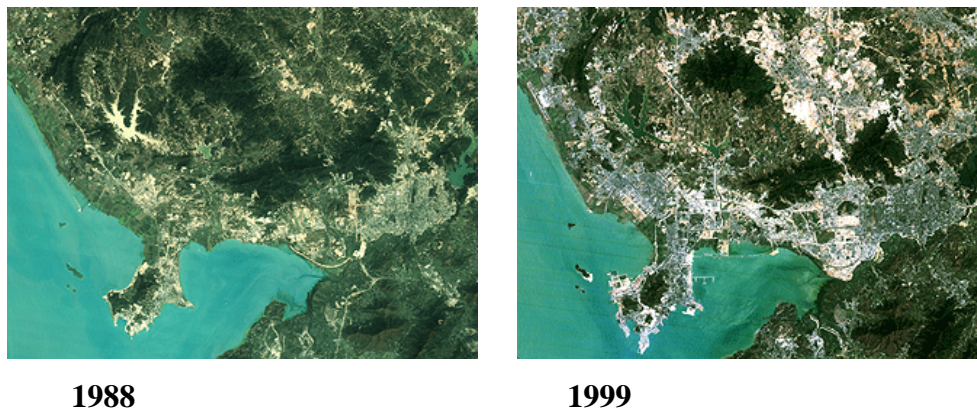


The population of Pucallpa has doubled eight times since 1943, with an average doubling time of 7.5 years and an average growth rate of over 9% per year. The surrounding rain forest habitat has been devastated as has been the fresh-water fishery of the Ucayali River, the main source of the Amazon (Hern. 1992).

On the other side of South America, São Paulo, the largest city in Brazil and the fourth largest in the world, has a metropolitan population currently estimated at nearly 19 million (UN, 2005). Its population grew from 1,500 in 1600 to 65,000 in 1890, to 580,000 in 1920, and to 2.3 million in 1950, making it third largest city in the world at that time. The population of São Paulo reached 6.5 million in 1965. By 2015, the UN projects that São Paulo will have 20.5 million people and will still be the fourth largest city in the world. At 35 million, Tokyo, the largest city, is nearly double the size of São Paulo.

China has experienced rapid urbanization since 1978. From 1978 to 2000, the number of small towns in China increased from 2,176 to 20,312, and the number of cities increased from 190 to 663. During this period, the proportion of urban population in China rose from 18% to 39%. Shenzhen, the fastest-growing city in China, once a small fishing village, was chosen as the site for a new port. It grew from ~ 0.1 million to ~ 7 million from 1978 to 2000. On average, the population of Shenzhen doubled every 3.6 years. Fig. 10 illustrates the changes in Shenzhen from 1988 through 1999.

Fig. 10. Shenzhen, China, 1988 – 1999 (from Zhou, et al, 2004)



The effect of such rapid urbanization in China has been the creation of a larger number of Urban Heat Island (UHI) effects, which have contributed directly to an effect on the average surface temperatures and evidence for a significant effect on climate (Zhou, et al, 2004).

In none of these examples is the growth of the city controlled, and there is no prospect for the control or limits of growth to be imposed by any entity. Batty calls this phenomenon a “spatial epidemic” in which the urban agglomeration reaches the stage of “self-organized criticality” (SOC) characterized by uncontrolled fractal growth independent of scale (Batty and Xie 1999; Batty, 2005, pp. 429, 441). The growth of metropolitan Buffalo, New York is given as an example.

The concept of “self-organized criticality” has been applied to malignant lesions, also, reflecting the fact that tumors display fractal growth independent of scale and independent of normal growth controls (Frigyesi, et al, 2003; Kitano, 2007; Kansal, et al, 2000; Dormann and Deutsch, 2002; Deutsch, 2001). Hallmarks of SOC include lack of any scale, in time as well as in space, fractal distributions, and power-law behavior, and these characteristics are found in malignant neoplasms (Waliszewski, et al, 2001).

The growth of cities at the scale we are now witnessing has its inexorable effect on the surrounding region, and many such large cities have their effect on global climate. Odum (1989) notes that cities are highly heterotrophic, parasitic ecosystems since they consume much more energy than they produce, and this magnifies the effect on the global ecosystem.

Molotch (1976) calls the modern city a “growth machine.” But it is not a machine. It is a biocultural adaptation to the survival needs of the human species and individual members of it. The “gigalopolis” (USGS, 2007) is one of the defining features of the human occupation of global ecosystems in the 21st century.

Complexity in cities and malignancies

The city is a complex adaptive system (Holland, 1995) that is subject to collapse in itself or within its heterotrophic network (Tainter, 1988). Camazine et al (2001:26) cite the growth of the human population as example of positive feedback in an SOC in which there is no antagonistic inhibitory mechanism (see also Bettencourt, et al, 2007). Such a system is inherently unstable (Margalef, 1968). We have shown evidence that human culture is the adaptive mechanism for eliminating negative feedback to human population growth and from ecological destruction (Hern, 1993). Cultural adaptations have eliminated predators, food scarcity, energy scarcity, and, with notable exceptions (Tainter, 1988), environmental limits to human population growth. Reflecting this is the fact that rapid urban growth has taken place in a context of unprecedented growth of the global human population, which has more than quadrupled in the century from the beginning of the twentieth century to the present – from 1.6 billion to more than 6.6 billion (US Census, 2007). The proportion of urban population, which was 220 million (13%) in 1900, has grown to 29% (732 million) in 1950 and 49% (3.2 billion) in 2005. By 2030, the 4.9 billion people living in urban agglomerations will represent 60% of the human population (UN, 2005). The expansion of cities has occurred at the expense of open countryside surrounding the cities in all cases.

Cancer as a metaphor

The comparison of the growth and forms of cities and cancer is obvious. Dismissing these comparisons as merely a “metaphor” without connection to fact or reality misses the point that metaphors summarize complex realities in ways that make them understandable (Bak, 1996). The metaphor must be tested against empirical fact, but if enough facts fall into the framework of the metaphor, it forms the basis for a hypothesis to be formulated and tested.

A typical modern city may not be a cancer. But it has all the characteristics of a cancer on the landscape. It is a complex (non-linear), dynamic (growing), topophagic (space-devouring) heterotrophic process that displays rapid, uncontrolled fractal growth, distant colonization, invasion and destruction of adjacent natural ecosystems, and de-differentiation.

In organismic pathology, at least two of these characteristics must be present to make a diagnosis of malignancy. The presence of any two of the first four classical characteristics define a neoplastic (new growth) tumor as malignant until proven otherwise (Anderson, 1961).

We have not included here another characteristic of malignant lesions, the excretion of toxic metabolites. This is true of cities (Wolman, 1965; Girardet, 1999; Brunner and Rechberger, 2002). Cancer cells resist apoptosis (programmed cell death) (Hanahan and Weinberg, 2000). It is one of the reasons that malignant tissues persist and grow until the supporting organism ceases to function. It dies. By comparison with the life spans of humans or other organisms, cities are immortal. They seldom cease entirely to exist.

Cancers show malignant progression – they become more malignant over time, especially in metastatic lesions (Ruddon, 1987; Hamada, et al, 1999; Morel-Chany, et al, 1985; Frigyesi, 2003). Human communities have shown increasingly faster growth rates over the past two hundred years, with new communities and cities often having the highest rates of growth, as in the examples of Brasilia, Shenzhen and Pucallpa.

Here are some features in common for malignancies and cities including those with respect to “self-organized criticality”:

- Rapid, uncontrolled fractal growth
- Invasion and destruction of adjacent normal (tissues/ecosystems)
- Metastasis (distant colonization)
- De-differentiation
- Progression (increasing rates of growth in new lesions/communities)
- Dissipative boundaries or structures (“...open systems in which there is a continual influx of energy or matter...structures emerge through interactions obeying non-linear kinetics.” [Camazine, et al, 2001:30])
- Emergent properties (“...a process by which a system of interacting subunits acquires qualitatively new properties that cannot be understood as the simple addition of their individual contributions...the system-level properties arise unexpectedly from nonlinear interactions among a system’s components..” [Camazine, et al, 2001:31])
- Scale invariance
- Dynamic (growing)
- Non-linear (complex)
- Topophagic (space-occupying or “space-devouring”)
- Apoptosis (resistance to programmed cell death / immortality of cities)
- Positive feedback loops without antagonizing inhibitory mechanisms
- “The defining characteristic of self-organizing systems is that their organization arises entirely from multiple interactions among components.” (Camazine, et al, 2001:20)

A hypothesis to be tested

The fractal dimension of urban agglomerations is often comparable to the fractal dimension of highly malignant solid tumors.

Aside from the obvious similarity of pattern with malignancy, the physical form and growth of the city can be compared mathematically with a malignant lesion through fractal

geometry. The same mathematical modeling is used to study fractal growth in malignant lesions. The testable (and refutable) null hypothesis is:

H₀: There is no similarity in the fractal dimension or fractal growth of a human community and a malignant lesion.

The mere fact that urban agglomerations and cancers share fractal properties does not in itself have meaning, even if it can be shown to be true. This important fact does, however, establish that there is at least one verifiable and mathematically comparable quality shared by cities and cancers. This is the primary hypothesis. If it is true, it raises an important question of why it is true and whether it is true because cities have other characteristics in common with malignancies.

Benguigui, et al (2000) ask “When and where is a city fractal?” But the next question could be, “*Why* is it fractal, and *why* does it display fractal growth?”

This primary hypothesis forms the premise for a more demanding secondary hypothesis that is explanatory and predictive: that cities (or urban agglomerations) exhibit all the major characteristics of a malignant neoplasm. These characteristics have been exhibited by all human communities over a long time beginning with the formation of communities in the Neolithic. In cancers, the latent period leading to fractal growth and self-organized criticality is also long in terms of the life span of the organism.

Purpose and implications of a hypothesis

These hypotheses explain the fractal dimension of cities, urban agglomerations and villages, and they explain the fact that it is scale-invariant. This is what permits the comparison between a malignant lesion that is one centimeter in diameter and a gigalopolis of 15-35 million people.

The secondary hypothesis can also be stated in the form of a null hypothesis: Cities have no characteristics in common with a malignant neoplasm. This is a falsifiable hypothesis. Only two characteristics of cancer are necessary for a diagnosis to be made.

One of the most compelling criticisms of this hypothesis is that it is a mere metaphor or analogy. But metaphor is an important tool in the formulation of scientific hypotheses, and metaphors help to distill and visualize the essential features of a phenomenon or hypothesis. And, as distinguished from cancer, one does not die from an analogy.

The purpose of a hypothesis is to describe reality and to predict events. These hypotheses explain some of the most important things we know about human communities. The next challenge is to determine, if possible, what is predicted by these hypotheses.

The consequences of cancer

Death of the host organism in a cancer occurs between the 37th and 40th doubling of the cell population (Tannock, 1992). The human population had doubled its numbers 32.5 times by 1999 (Hern, 1999), but global energy use by human beings has doubled 36 times, is doubling at the rate of about 2.3% per year (US Energy Information Administration, 1998), and will increase 57% from 2004 to 2030 (US EIA, 2007). Since cities are entirely heterotrophic and depend on external energy sources, this increase in energy use and resulting global climate change is directly related to the inexorable, malignant growth of universally heterotrophic cities that are expanding without limit or control.

Prognosis

A prognosis is not the same as a prediction, which has more certainty than a prognosis. But the prognosis of an untreated cancer is severe. Cancers continue to grow in the face of host starvation until the host – the supporting organism – ceases to function. It dies. The supporting organism, in the case of human beings and the planet Earth, is the biosphere. At the present, the biosphere is particularly hospitable to all forms of life from the submicroscopic to the largest creatures. Anthropogenic modification of the biosphere through a long series of malignant maladaptations may not destroy the biosphere, but it may alter the biosphere to the point that it can no longer support large, oxygen-consuming organisms. A soaring average global temperature with undamped oscillations (Margalef, 1968) may extinguish most plants, as well. From the point of view of human survival, the host organism will have died.

If this hypothesis is correct, that urban settlements of all kinds, and cities in particular, are malignant lesions or phenomena on the planet Earth, the conclusion of this process may not be far in the future.

Notes

1. Mandelbrot (1983), p. 25.
2. Mandelbrot defines a fractal as "... a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension." p.15.

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